



Figure 11-16 Graphic arch analysis. (From BIA Technical Note 31.)

this point, draw a line parallel to the line 7-2 until it intersects the line of action of 2-3. Through this point, draw a line parallel to the line 7-3, and so on, and complete the polygon in this manner. If the polygon lies completely within the middle third of the arch section, the arch is stable. For a uniformly distributed load, the equilibrium polygon, which coincides with the line of resistance, will normally fall within this section, but for other loading conditions it may not.

The eccentricity of the voussoir reactions will produce stresses which differ from the axial stress  $H/A$ , where  $A$  is the cross-sectional area of the arch ( $A = bd$ ). These stresses are determined by the formula

$$f_m = \frac{H}{bd} \pm \frac{6He}{bd} \tag{11.1}$$

where  $f_m$  = maximum compressive stress in the arch, psi

$H$  = horizontal thrust, lb

$b$  = thickness of the arch, in.

$d$  = depth of the arch, in.

$e$  = perpendicular distance between the arch axis and the line of action of the horizontal thrust, in.

Maximum allowable compressive stresses in an arch are determined on the basis of the compressive strength of the units and mortar, and are governed by the same code requirements as other masonry construction (see Chapter 12).

A number of mathematical formulas have been developed for the design of minor arches. Among the structural considerations are three methods of failure of unreinforced masonry arches: (1) by rotation of one section of the arch about the edge of a joint; (2) by the sliding of one section of the arch on another or on the skewback; and (3) by crushing the masonry.

### 11.2.3 Rotation

The assumption that the equilibrium polygon lies entirely within the middle third of the arch section precludes the rotation of one section of the arch about the edge of a joint or the development of tensile stresses in either the intrados or extrados. For conditions other than evenly distributed uniform loads, where the polygon may fall outside the middle third, however, this method of failure should be considered.

### 11.2.4 Sliding

The coefficient of friction between the units of a masonry arch is at least 0.60 without considering the additional resistance to sliding resulting from the bond between the mortar and the masonry units. This corresponds to an angle of friction of approximately  $31^\circ$ . If the angle between the line of resistance and a line perpendicular to the joint between sections is less than the angle of friction, the arch is stable against sliding. This angle may be determined graphically as shown above. For minor segmental arches, the angle between the line of resistance and a line perpendicular to the joint is greatest at the skewback. This is also true for jack arches if the joints are radial about a center at the intersection of the planes of the skewbacks. However, if the joints are not radial about this center, each joint should be investigated separately for resistance to sliding. This can be most easily accomplished by constructing an equilibrium polygon.

### 11.2.5 Crushing

A segmental arch is one whose curve is circular but is less than a semicircle. The minimum recommended rise for a segmental arch is 1 in. per foot of span. The horizontal thrust developed depends on the span, depth, and rise of the arch.

The graph in *Fig. 11-17* identifies thrust coefficients ( $H/W$ ) for segmental arches subject to uniform loads over the entire span. Once the thrust coefficient is determined for a particular arch, the horizontal thrust ( $H$ ) may be determined as the product of the thrust coefficient and the total load ( $W$ ). To determine the proper thrust coefficient, first determine the characteristics  $S/r$  and  $S/d$  of the arch, where  $S$  is the clear span in feet,  $r$  is the rise of the soffit in feet, and  $d$  is the depth of the arch in feet. If the applied load is triangular or concentrated, the same method may be used, but the coefficient  $H/W$  is increased by one-third for triangular loading and doubled for concentrated loads.

Once the horizontal thrust has been determined, the *maximum compressive stress* in the masonry is determined by the formula

$$f_m = \frac{2H}{bd} \quad (11.2)$$

This value is twice the axial compressive stress on the arch due to the load  $H$ , because the horizontal thrust is located at the third point of the arch depth.

The common rule for *jack arches* is to provide a skewback ( $K$  measured horizontally; see *Fig. 11-15*) of  $\frac{1}{2}$  in. per foot of span for each 4 in. of arch depth. Jack arches are commonly constructed in depths of 8 and 12 in. with a camber of  $\frac{1}{8}$  in. per foot of span. To determine the *horizontal thrust* at the spring line for jack arches, the following formulas may be used: